# Chapter 2

Optical Fibers: Structures, Waveguiding & Fabrication

Theories of Optics

- Light is an electromagentic phenomenon described by the same theoretical principles that govern all forms of electromagnetic radiation. **Maxwell's equations** are in the heart of electromagnetic theory & is fully successful in providing treatment of light propagation. **Electromagnetic optics** provides the most complete treatment of light phenomena in the context of **classical optics.**

- Turning to phenomena involving the interaction of **light & matter,** such as **emission** & **absorptio**n of light, **quantum theory** provides the successful explanation for light-matter interaction. These phenomena are described by quantum electrodynamics which is the marriage of electromagnetic theory with quantum theory. For optical phenomena, this theory also referred to as **quantum optics.** This theory provides an explanation of virtually all optical phenomena.

- In the context of classical optics, electromagentic radiation propagates in the form of two mutually coupled vector waves, an **electric field-wave** & **magnetic field wave**. It is possible to describe many optical phenomena such as diffraction, by **scalar** wave theory in which light is described by a single scalar wavefunction. This approximate theory is called scalar wave optics or simply **wave optics**. When light propagates through & around objects whose dimensions are much greater than the optical wavelength, the wave nature of light is not readily discerned, so that its behavior can be adequately described by rays obeying a set of geometrical rules. This theory is called **ray optics.** Ray optics is the limit of wave optics when the wavelength is very short.

Quantum Optics

Electromagnetic Optics

Wave Optics

Ray Optics
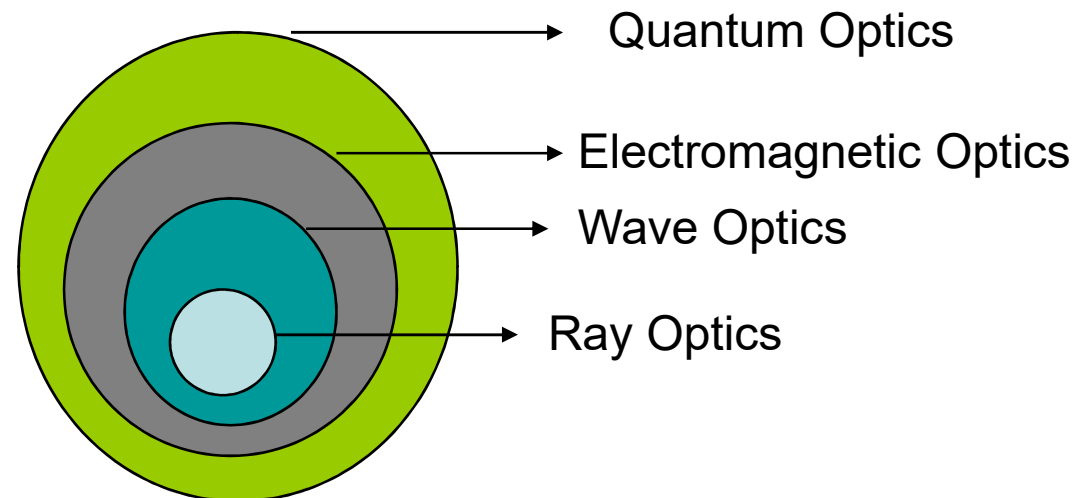
Engineering Model

- In engineering discipline, we should choose the appropriate & easiest physical theory that can handle our problems. Therefore, specially in this course we will use different optical theories to describe & analyze our problems. In this chapter we deal with optical transmission through fibers, and other optical waveguiding structures. Depending on the structure, we may use ray optics or electromagnetic optics, so we begin our discussion with a brief introduction to electromagnetic optics, ray optics & their fundamental connection, then having equipped with basic theories, we analyze the propagation of light in the optical fiber structures.

Electromagnetic Optics

- Electromagnetic radiation propagates in the form of two mutually coupled **vector** waves, an **electric field wave** & a **magnetic field wave**. Both are vector functions of position & time.

- In a <u>source-free, linear, homogeneous, isotropic & non-dispersive media, such as free space,</u> these electric & magnetic fields satisfy the following partial differential equations, known as **Maxwell' equations:**

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \qquad \text{[2-1]}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \qquad \text{[2-2]}$$

$$\nabla \cdot \vec{E} = 0 \qquad \text{[2-3]}$$

$$\nabla \cdot \vec{H} = 0 \qquad \text{[2-4]}$$

- In Maxwell's equations, E is the electric field expressed in [V/m], H is the magnetic field expressed in [A/m].

$$\varepsilon \,[\text{F/m}] : \text{Electric permittivity}$$

$$\mu \,[\text{H/m}] : \text{Magnetic permeability}$$

$$\nabla \cdot : \text{is divergence operation}$$

$$\nabla \times : \text{is curl operation}$$

- The solution of Maxwell's equations in free space, through the **wave equation,** can be easily obtained for **monochromatic** electromagnetic wave. All electric & magnetic fields are harmonic functions of time of the same frequency. Electric & magnetic fields are perpendicular to each other & both perpendicular to the direction of propagation, k, known as **transverse wave (TEM)**. *E, H* & *k* form a set of orthogonal vectors.

Electromagnetic Plane wave in Free space
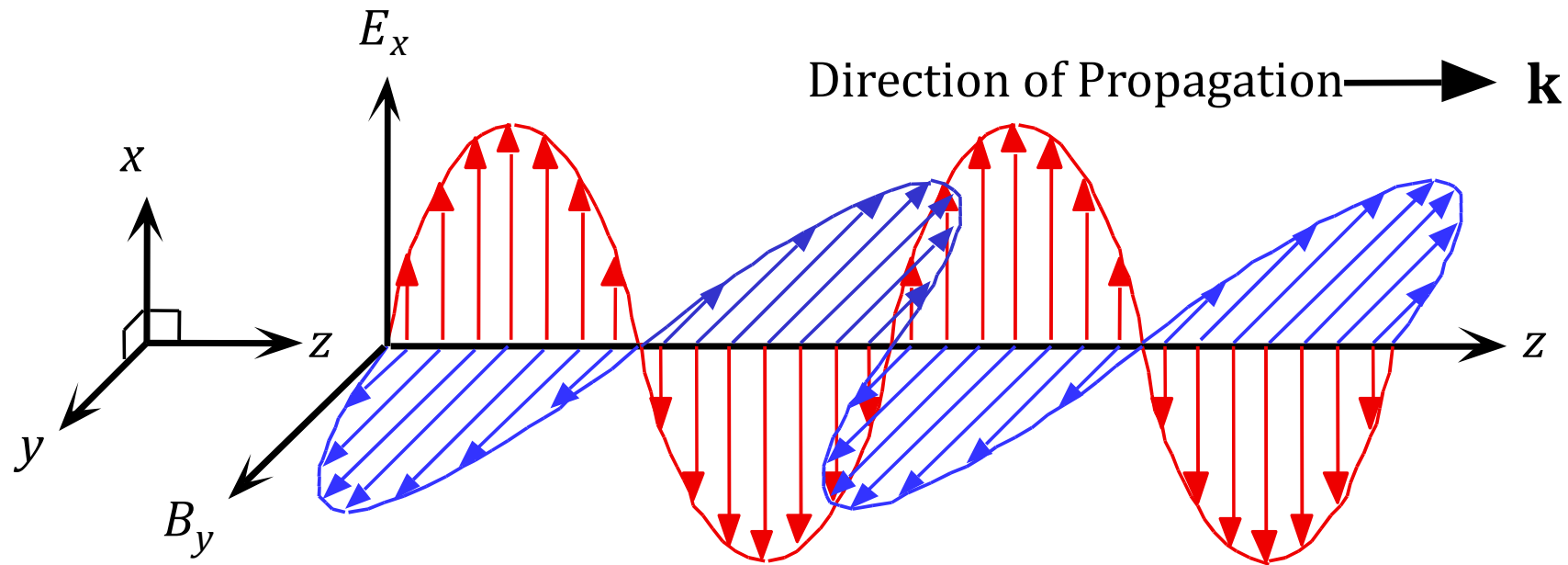
An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and the direction of propagation, z.

S.O.Kasap, optoelectronics and Photonics Principles and Practices, prentice hall, 2001

Linearly Polarized Electromagnetic Plane wave

$$\vec{E} = \mathrm{e}_x E_{0x} \cos(\omega t - kz) \qquad \text{[2-5]}$$

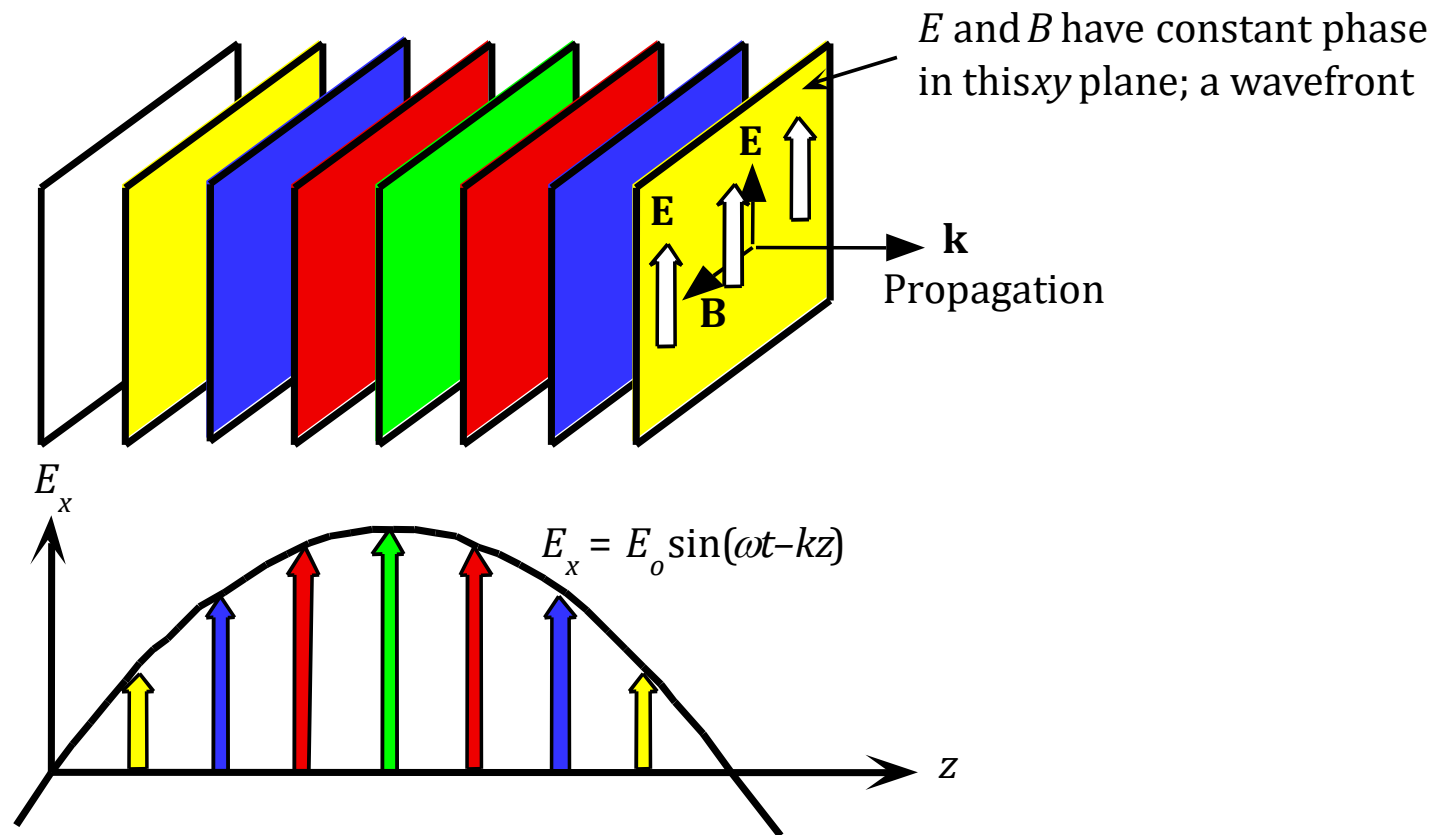$$\vec{H} = \mathrm{e}_y H_{0y} \cos(\omega t - kz) \qquad \text{[2-6]}$$

where :

$\omega = 2\pi f$: Angular frequency [rad/m] $\qquad$ [2-7]

k : Wavenumber or wave propagation constant [1/m]

$$\lambda = \frac{2\pi}{k} : \text{Wavelength [m]}$$

$$\eta = \frac{E_{0x}}{H_{0y}} = \sqrt{\frac{\mu}{\varepsilon}} \; [\Omega]: \quad \text{intrinsic (wave) impedance} \qquad \text{[2-8]}$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \, [\mathrm{m/s}]: \quad \text{velocity of wave propagation} \qquad \text{[2-9]}$$

E and *B* have constant phase in this *xy* plane; a wavefront

E

E

B

k

Propagation

E_x

$E_x = E_o \sin(\omega t - kz)$

z

A plane EM wave travelling along *z*, has the same E_x (or B_y) at any point in a given *xy* plane. All electric field vectors in a given *xy* plane are therefore in phase. The *xy* planes are of infinite extent in the *x* and *y* directions.

Wavelength & free space

- **Wavelength** is the distance over which the phase changes by 2π .

$$\lambda = \frac{v}{f} \qquad\qquad \text{[2-10]}$$

- In vacuum (free space):

$$\varepsilon_0 = \frac{10^{-9}}{36\pi} \text{ [F/m]} \qquad\qquad \mu_0 = 4\times10^{-7} \text{ [H/m]}$$

[2-11]

$$v = c \cong 3\times10^{8} \text{ m/s} \qquad\qquad \eta_0 = 120\pi \text{ [}\Omega\text{]}$$

EM wave in Media

- **Refractive index** of a medium is defined as:

$$n = \frac{c}{v} = \frac{\text{velocity of light (EM wave) in vacuum}}{\text{velocity of light (EM wave) in medium}} = \sqrt{\frac{\mu \varepsilon}{\mu_0 \varepsilon_0}} = \sqrt{\mu_r \varepsilon_r}$$

[2-12]

μ_r ： Relative magnetic permeability

ε_r ： Relative electric permittivity

- For non-magnetic media $(\mu_r = 1)$:

$$n = \sqrt{\varepsilon_r}$$

[2-13]

Intensity & power flow of TEM wave

- The poynting vector $S = \dfrac{1}{2}\vec{E} \times \vec{H}^{*}$ for TEM wave is parallel to the

  wavevector **k** so that the power flows along in a direction normal to the
  wavefront or parallel to **k**. The magnitude of the poynting vector is the
  intensity of TEM wave as follows:

$$I = \frac{\left| E_0 \right|^2}{2\eta} \quad [\text{W/m}^2] \qquad\qquad [2\text{-}14]$$

Connection between EM wave optics & Ray optics

According to wave or physical optics viewpoint, the EM waves radiated by a small optical source can be represented by a train of spherical wavefronts with the source at the center. A **wavefront** is defined as the locus of all points in the wave train which exhibit the same phase. Far from source wavefronts tend to be in a plane form. Next page you will see different possible phase fronts for EM waves.

When the wavelength of light is much smaller than the object, the wavefronts appear as straight lines to this object. In this case the light wave can be indicated by a **light ray,** which is drawn perpendicular to the phase front and parallel to the Poynting vector, which indicates the flow of energy. Thus, large scale optical effects such as reflection & refraction can be analyzed by simple geometrical process called **ray tracing**. This view of optics is referred to as **ray optics** or **geometrical optics**.
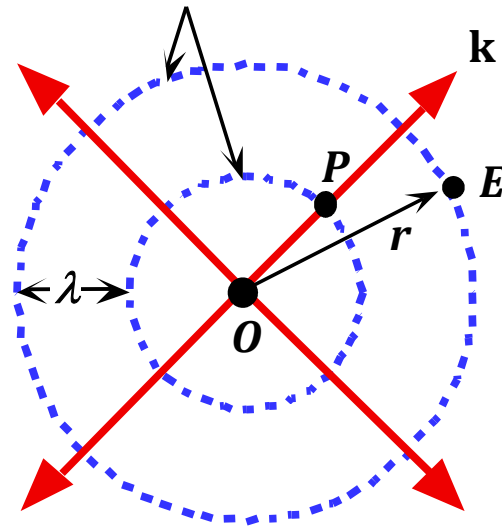
Wave fronts
(constant phase surfaces)

rays

λ λ
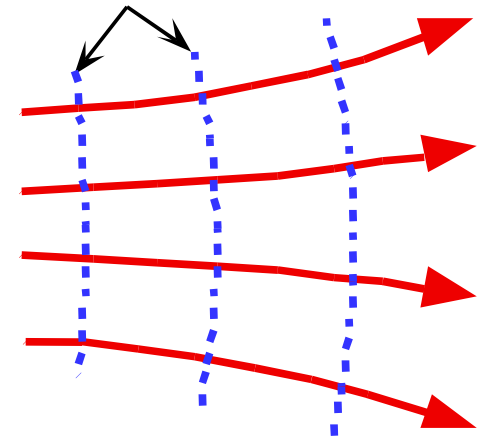
P

k

z
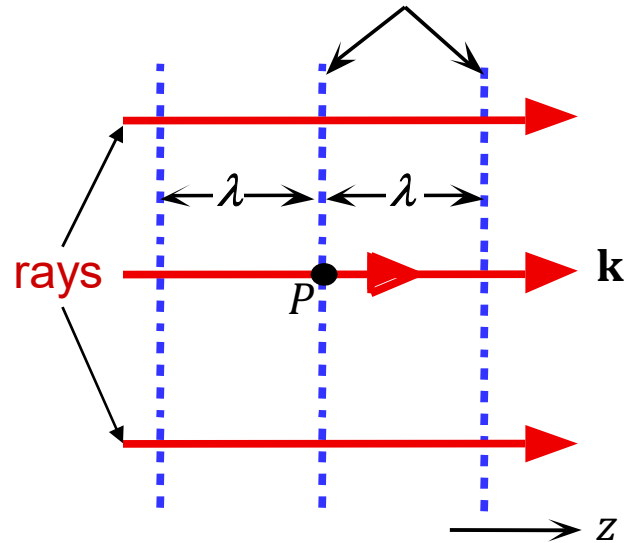
A perfect plane wave

(a)

Wave fronts

k

P E

r

λ

O

A perfect spherical wave

(b)

Wave fronts

A divergent beam

(c)

Examples of possible EM waves

S.O.Kasap, optoelectronics and Photonics Principles and Practices, prentice hall, 2001

General form of linearly polarized plane waves

Any two orthogonal plane waves
Can be combined into a linearly
Polarized wave. Conversely, any
arbitrary linearly polarized wave
can be resolved into two
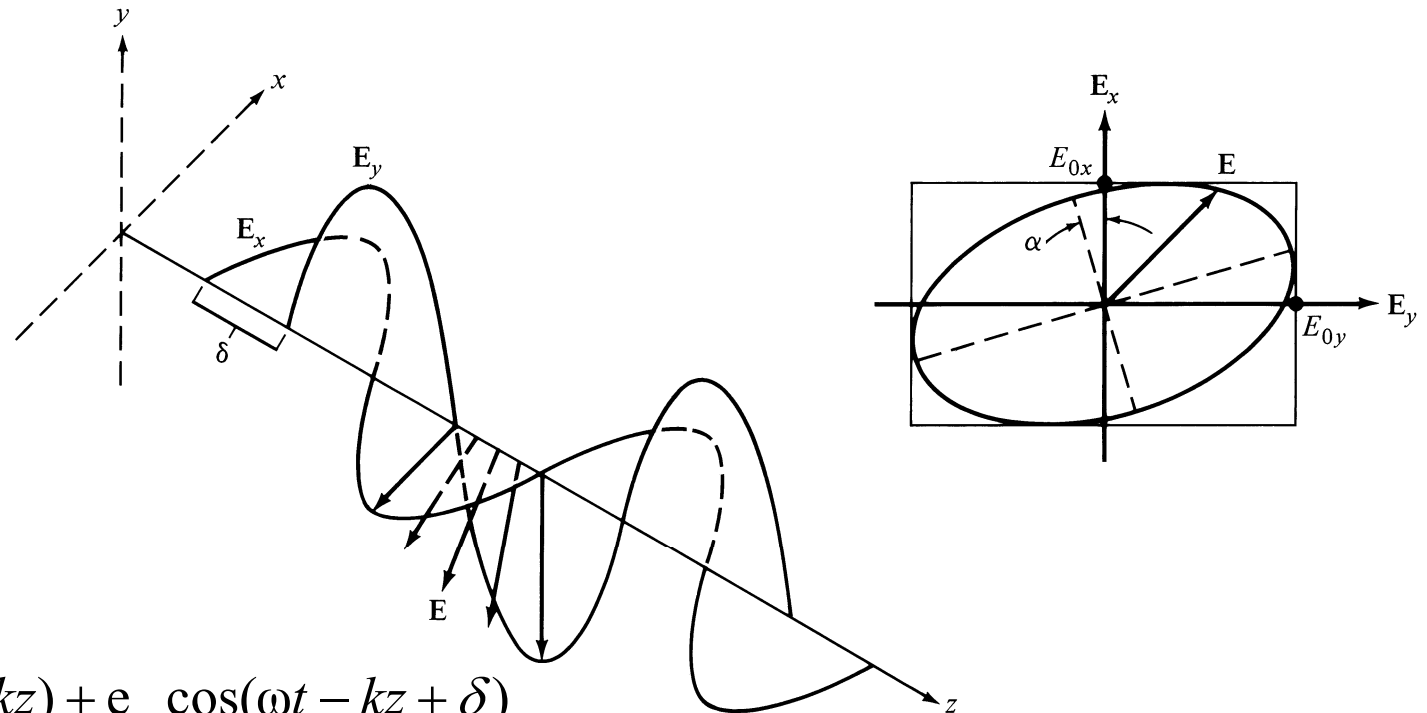independent Orthogonal plane
waves that are in phase.
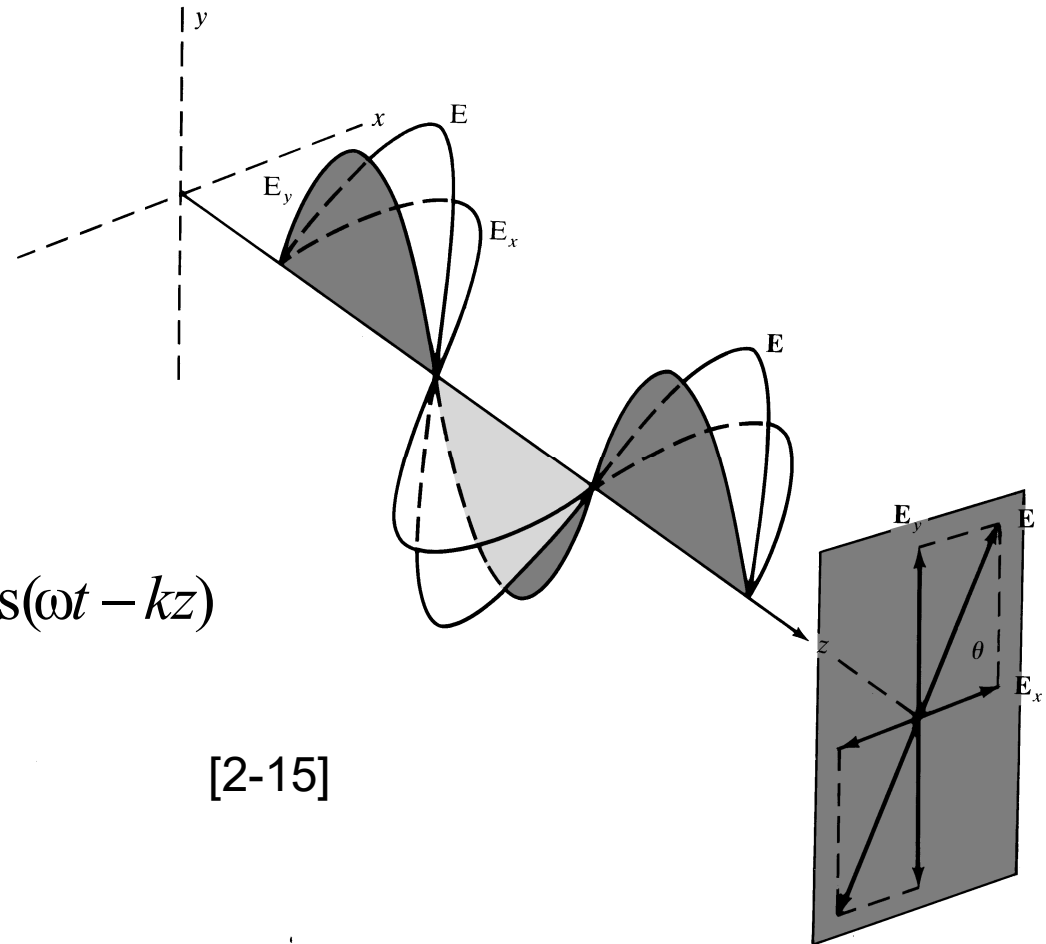
$$\vec{E} = e_x E_{0x} \cos(\omega t - kz) + e_y E_{0y} \cos(\omega t - kz)$$

$$E = \left| \vec{E} \right| = \sqrt{E_{0x}^2 + E_{0y}^2}$$

$$\theta = \tan^{-1}(\frac{E_{0y}}{E_{0x}})$$

[2-15]

Elliptically Polarized plane waves

$$\vec{E} = \mathbf{e}_x E_x + \mathbf{e}_y \mathbf{E}_y$$

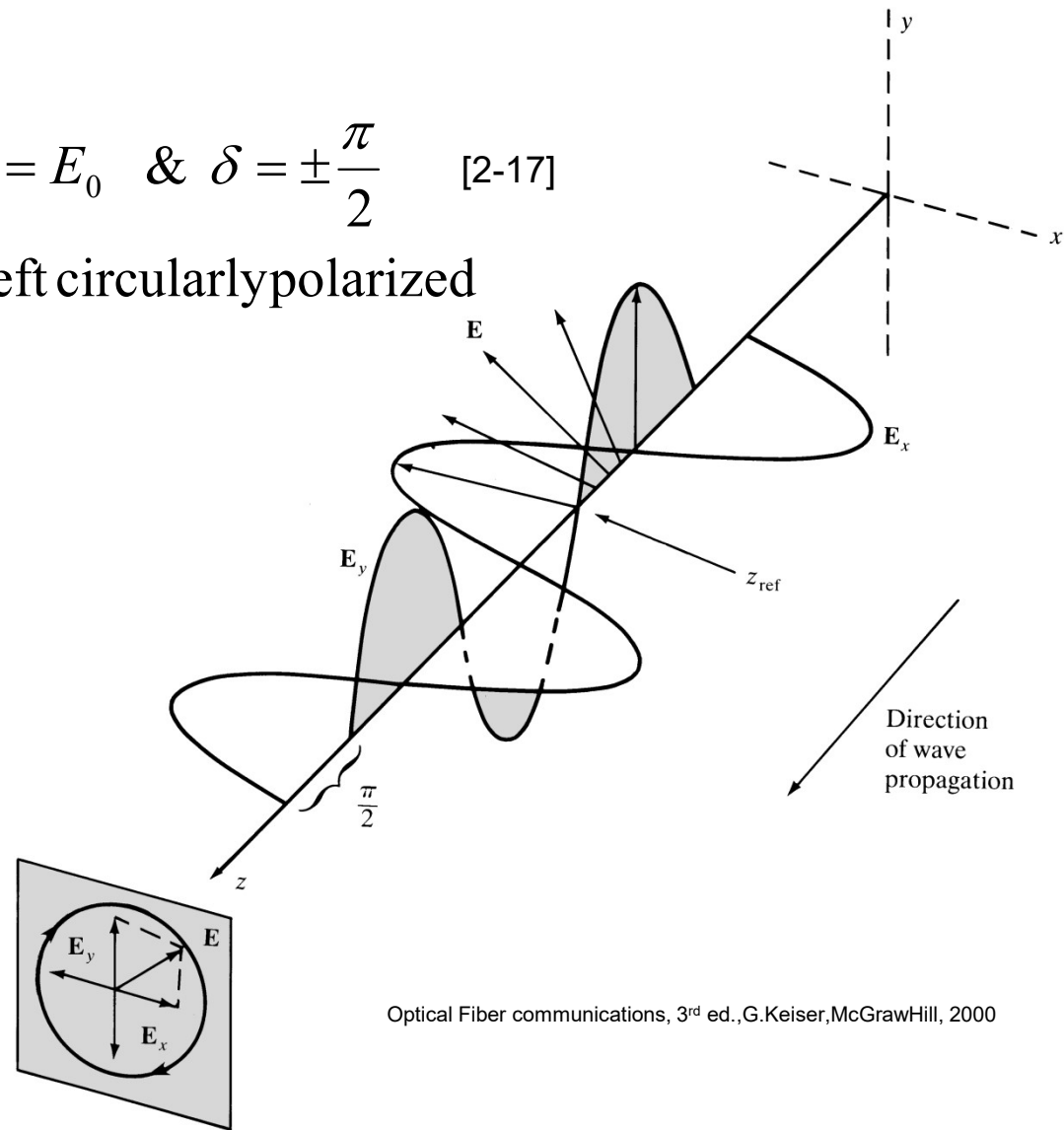$$= \mathbf{e}_x E_{0x} \cos(\omega t - kz) + \mathbf{e}_y \cos(\omega t - kz + \delta)$$

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\delta = \sin^2\delta \qquad \text{[2-16]}$$

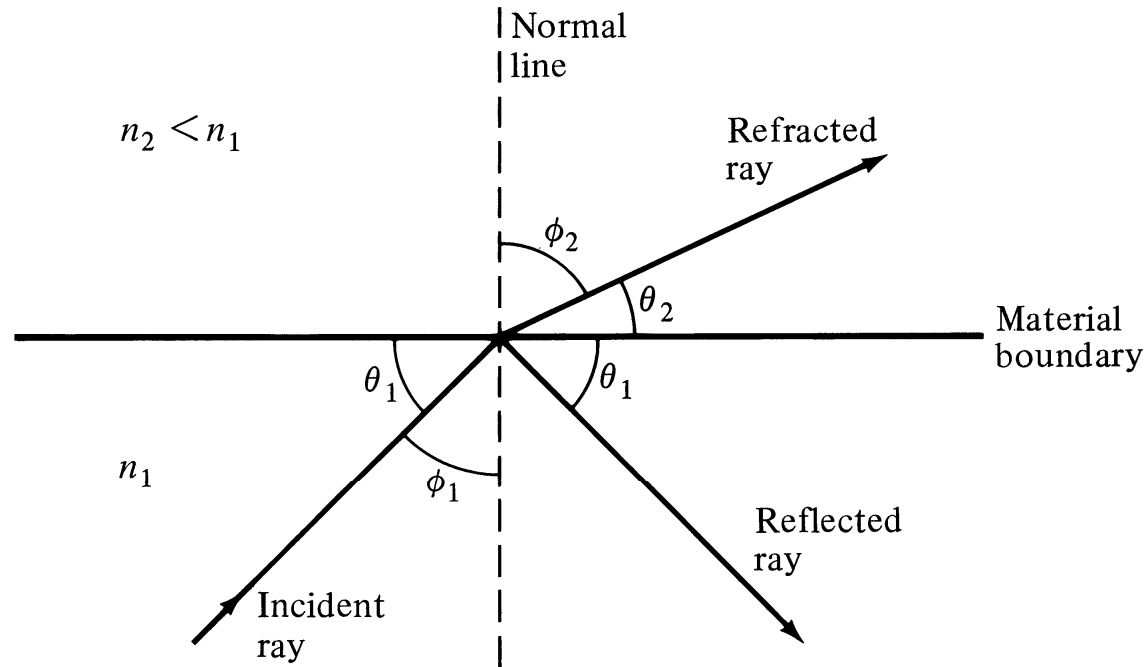$$\tan(2\alpha) = \frac{2E_{0x}E_{0y}\cos\delta}{E_{0x}{}^2 - E_{0y}{}^2}$$

Circularly polarized waves

Circular polarization : $E_{0x} = E_{0y} = E_0$ & $\delta = \pm\dfrac{\pi}{2}$ [2-17]

$+$: right circularly polarized, $-$: left circularly polarized

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000
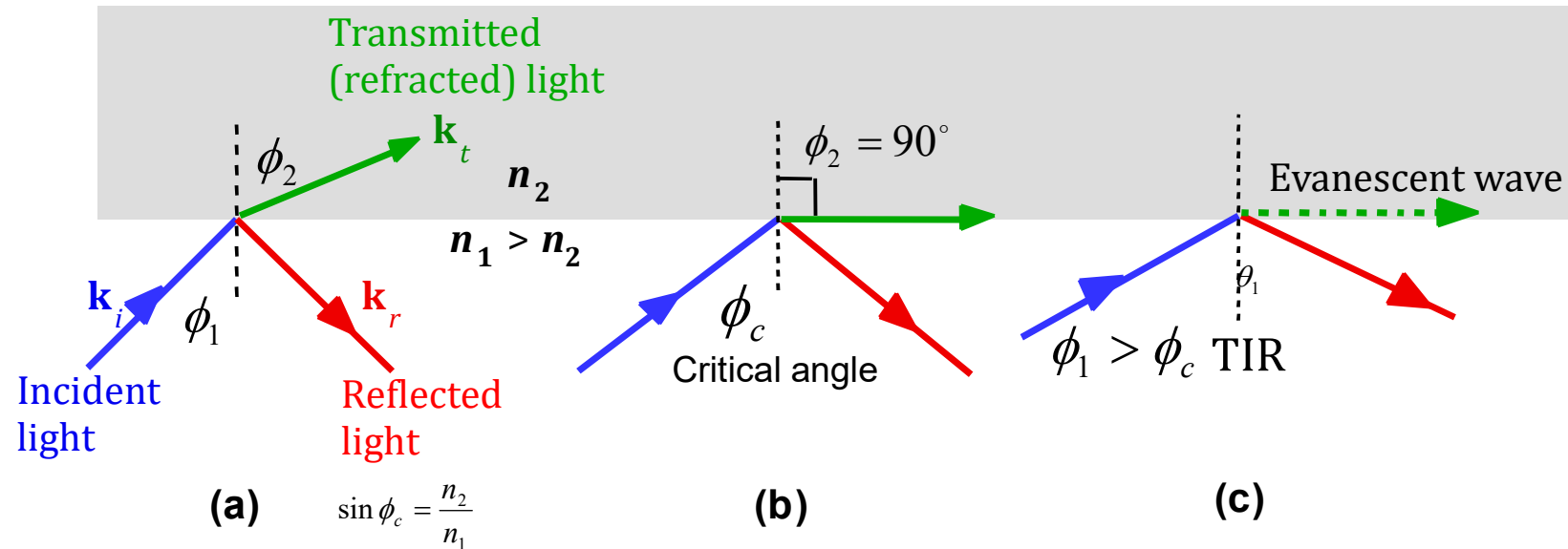
Laws of Reflection & Refraction

Reflection law: angle of incidence=angle of reflection

Snell's law of refraction:

$$n_1 \sin \phi_1 = n_2 \sin \phi_2 \qquad \text{[2-18]}$$

Total internal reflection, Critical angle

(a) $\sin \phi_c = \frac{n_2}{n_1}$ (b)   (c)

Light wave travelling in a more dense medium strikes a less dense medium. Depending the incidence angle with respect to, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected (a) $\phi_1 < \phi_c$ (b) $\phi_1 = \phi_c$ (c) $\phi_1 > \phi_c$ and total internal reflection (TIR).

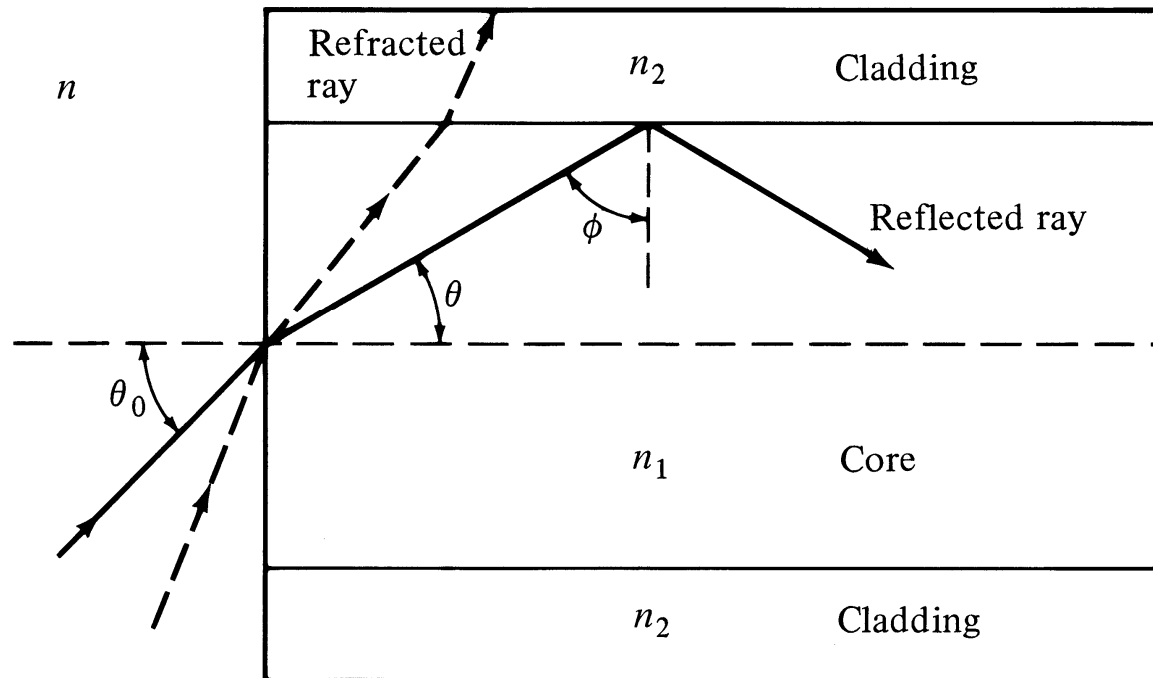$$\sin \phi_c = \frac{n_2}{n_1} \qquad \text{[2-19]}$$

Phase shift due to TIR

- The totally reflected wave experiences a phase shift however which is given by:

$$\tan\frac{\delta_N}{2} = \frac{\sqrt{n^2\cos^2\theta_1 - 1}}{n\sin\theta_1}; \quad \tan\frac{\delta_p}{2} = \frac{n\sqrt{n^2\cos^2\theta_1 - 1}}{\sin\theta_1} \qquad [2\text{-}20]$$

$$n = \frac{n_1}{n_2}$$

- Where (*p,N*) refer to the electric field components parallel or normal to the plane of incidence respectively.

Optical waveguiding by TIR: Dielectric Slab Waveguide

Propagation mechanism in an ideal step-index optical waveguide.

Launching optical rays to slab waveguide

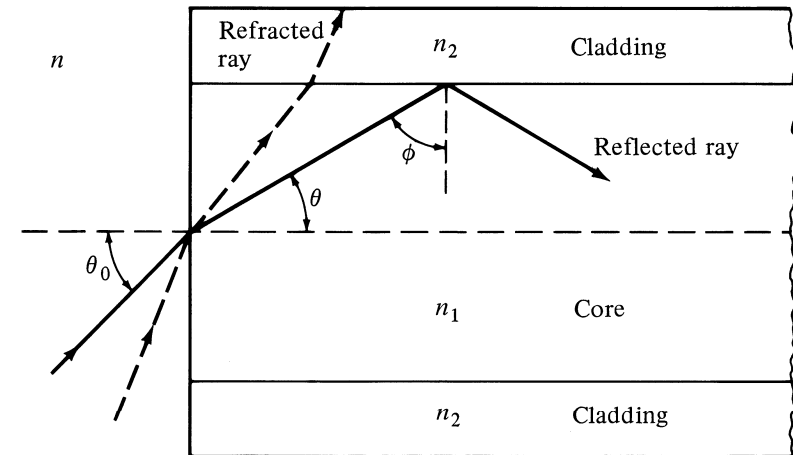$$\sin \phi_{min} = \frac{n_2}{n_1}; \text{ minimum angle that supports TIR}$$

[2-21]

Maximum entrance angle, $\theta_{0\,max}$ is found from the Snell's relation written at the fiber end face.

$$n \sin \theta_{0\,max} = n_1 \sin \theta_c = \sqrt{n_1^2 - n_2^2}$$ [2-22]
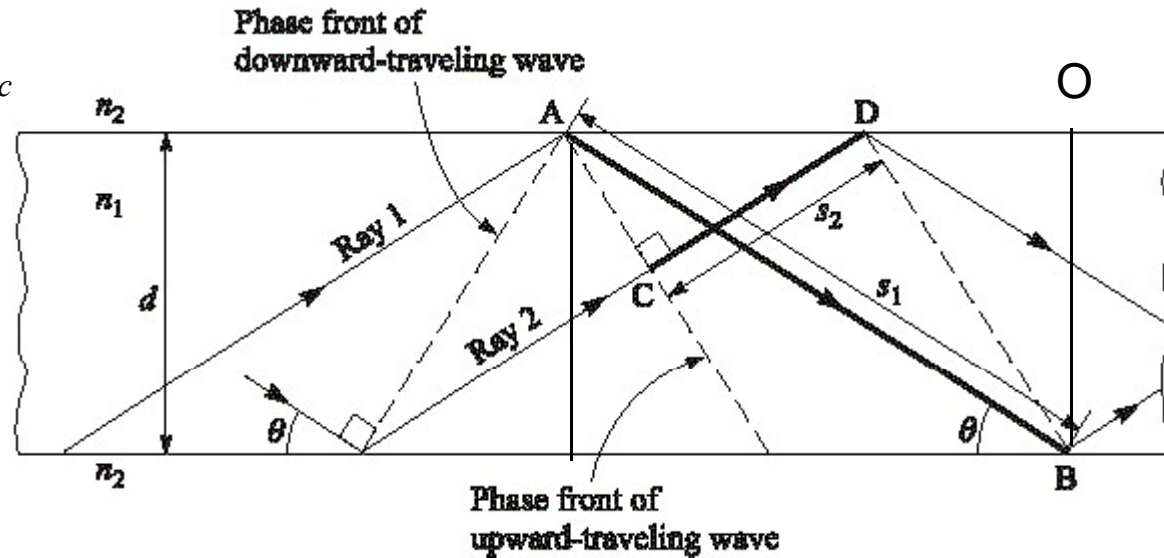
Numerical aperture:

$$NA = n \sin \theta_{0\,max} = \sqrt{n_1^2 - n_2^2} \approx n_1 \sqrt{2\Delta}$$ [2-23]

$$\Delta = \frac{n_1 - n_2}{n_1}$$ [2-24]

Optical rays transmission through dielectric slab waveguide

$$n_1 > n_2 \, ; \theta < \theta_c = \frac{\pi}{2} - \phi_c$$

Phase front of downward-traveling wave

Ray 1

Ray 2

Phase front of upward-traveling wave

For TE-case, when electric waves are normal to the plane of incidence
θ must be satisfied with following relationship:

$$\tan\left(\frac{\pi n_1 d \sin\theta}{\lambda} - \frac{m\pi}{2} \right) = \left[\frac{\sqrt{n_1^2 \cos^2\theta - n_2^2}}{n_1 \sin\theta} \right] \qquad \text{[2-25]}$$

Note

- **Home work 2-1**) Find an expression for θ ,considering that the electric field component of optical wave is parallel to the plane of incidence (TM-case).

- As you have seen, the polarization of light wave down the slab waveguide changes the condition of light transmission. Hence we should also consider the EM wave analysis of EM wave propagation through the dielectric slab waveguide. In the next slides, we will introduce the fundamental concepts of such a treatment, without going into mathematical detail. Basically we will show the result of solution to the Maxwell's equations in different regions of slab waveguide & applying the boundary conditions for electric & magnetic fields at the surface of each slab. We will try to show the connection between EM wave and ray optics analyses.

EM analysis of Slab waveguide

- For each particular angle, in which light ray can be faithfully transmitted along slab waveguide, we can obtain one possible propagating wave solution from a Maxwell's equations or **mode.**

- The modes with electric field perpendicular to the plane of incidence (page) are called **TE** (Transverse Electric) and numbered as: $TE_0, TE_1, TE_2, ...$

  Electric field distribution of these modes for 2D slab waveguide can be expressed as:
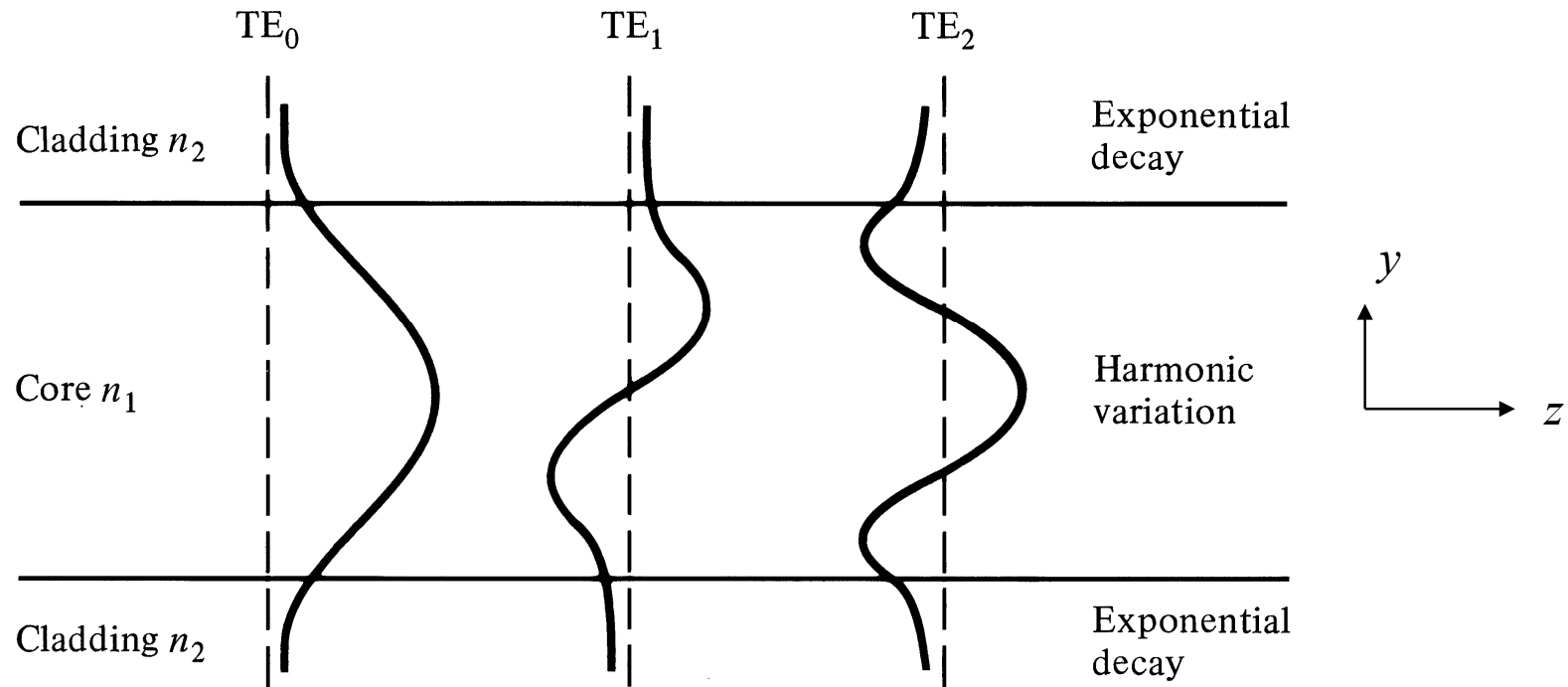
$$\vec{E}_m(x, y, z, t) = e_x f_m(y) \cos(\omega t - \beta_m z) \qquad [2\text{-}26]$$

$$m = 0, 1, 2, 3 \text{ (mode number)}$$

wave transmission along slab waveguides, fibers & other type of optical waveguides can be fully described by time & z dependency of the mode:

$$\cos(\omega t - \beta_m z) \qquad \text{or} \qquad e^{j(\omega t - \beta_m z)}$$

TE modes in slab waveguide

$$\vec{E}_m(x,y,z,t) = e_x f_m(y)\cos(\omega t - \beta_m z)$$

$$m = 0,1,2,3 \text{ (mode number)}$$

Modes in slab waveguide

- The order of the mode is equal to the # of field zeros across the guide. The order of the mode is also related to the angle in which the ray congruence corresponding to this mode makes with the plane of the waveguide (or axis of the fiber). **The steeper the angle, the higher the order of the mode**.

- For higher order modes the fields are distributed more toward the edges of the guide and penetrate further into the cladding region.

- **Radiation modes** in fibers are not trapped in the core & guided by the fiber but they are still solutions of the Maxwell' eqs. with the same boundary conditions. These infinite continuum of the modes results from the optical power that is outside the fiber acceptance angle being refracted out of the core.

- In addition to bound & refracted (radiation) modes, there are **leaky modes** in optical fiber. They are partially confined to the core & attenuated by continuously radiating this power out of the core as they traverse along the fiber (results from Tunneling effect which is quantum mechanical phenomenon.) A mode remains guided as long as $n_2 k < \beta < n_1 k$

Optical Fibers: Modal Theory (Guided or Propagating modes) & Ray Optics Theory
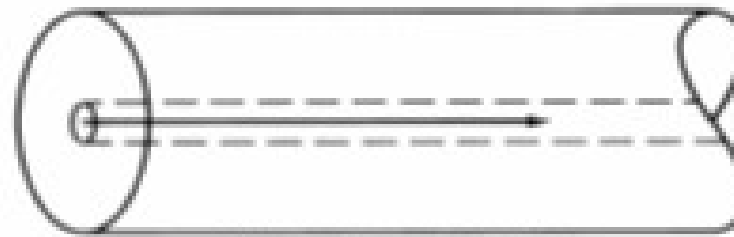

Core Cladding Buffer coating
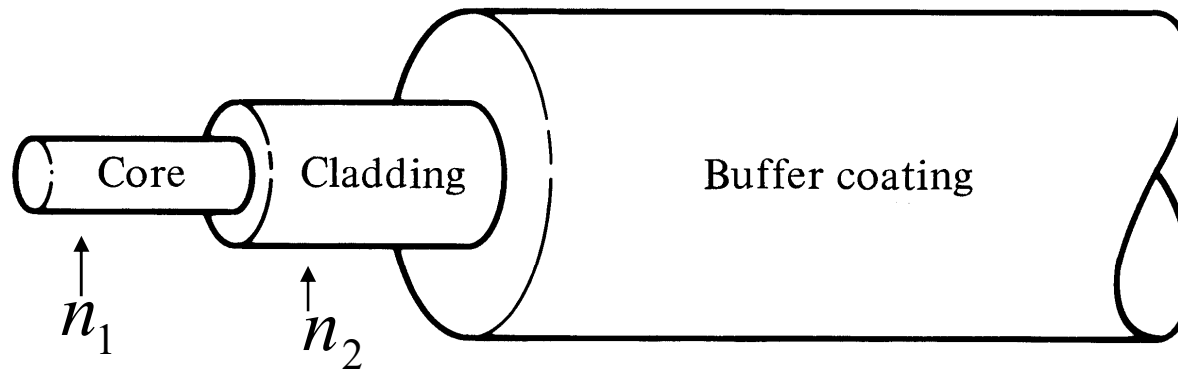
n_1 n_2

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

$$n_1 > n_2$$

Step Index Fiber

Index Profile

Fiber Cross Section and Ray Paths

Monomode step-index fiber

Modal Theory of Step Index fiber

- General expression of EM-wave in the circular fiber can be written as:

$$\vec{E}(r,\phi,z,t) = \sum_m A_m \vec{E}_m(r,\phi,z,t) = \sum_m A_m \vec{U}_m(r,\phi)e^{j(\omega t - \beta_m z)}$$

$$\vec{H}(r,\phi,z,t) = \sum_m A_m \vec{H}_m(r,\phi,z,t) = \sum_m A_m \vec{V}_m(r,\phi)e^{j(\omega t - \beta_m z)}$$

[2-27]

- Each of the characteristic solutions $\vec{E}_m(r,\phi,z,t)$ & $\vec{H}_m(r,\phi,z,t)$ is called mth **mode** of the optical fiber.

- It is often sufficient to give the E-field of the mode.

$$\vec{U}_m(r,\phi)e^{j(\omega t - \beta_m z)} \qquad m = 1,2,3...$$

- The modal field distribution, $\vec{U}_m(r,\phi)$, and the mode propagation constant, β_m are obtained from solving the Maxwell's equations subject to the boundary conditions given by the cross sectional dimensions and the dielectric constants of the fiber.

- Most important characteristics of the EM transmission along the fiber are determined by the mode propagation constant, $\beta_m(\omega)$, which depends on the mode & in general varies with frequency or wavelength. <u>This quantity is always between the plane propagation constant (wave number) of the core & the cladding media</u> .
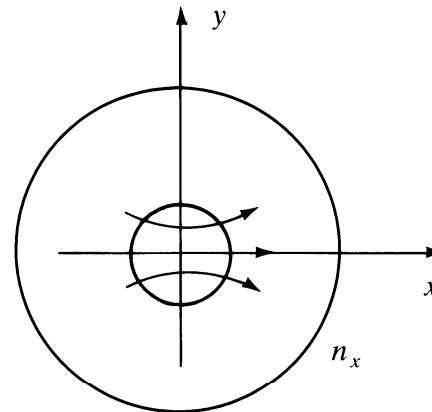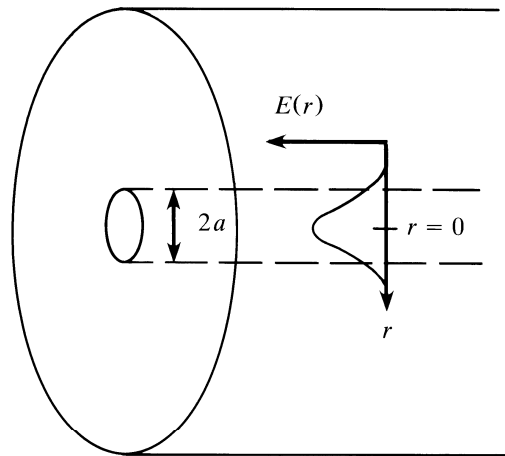
$$n_2 k < \beta_m(\omega) < n_1 k \qquad \text{[2-28]}$$

- At each frequency or wavelength, there exists only a finite number of guided or propagating modes that can carry light energy over a long distance along the fiber. Each of these modes can propagate in the fiber only if the frequency is above the **cut-off frequency**, ω_c, (or the source wavelength is smaller than the cut-off wavelength) obtained from cut-off condition that is:

$$\beta_m(\omega_c) = n_2 k \qquad \text{[2-29]}$$

- To minimize the signal distortion, the fiber is often operated in a **single mode** regime. In this regime only the lowest order mode (fundamental mode) can propagate in the fiber and all higher order modes are under cut-off condition (non-propagating).

- **Multi-mode** fibers are also extensively used for many applications. In these fibers many modes carry the optical signal collectively & simultaneously.
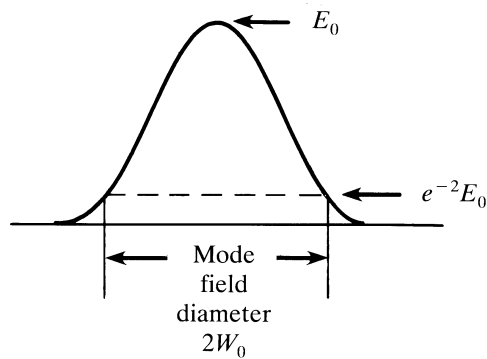
Fundamental Mode Field Distribution

$E(r)$

$2a$
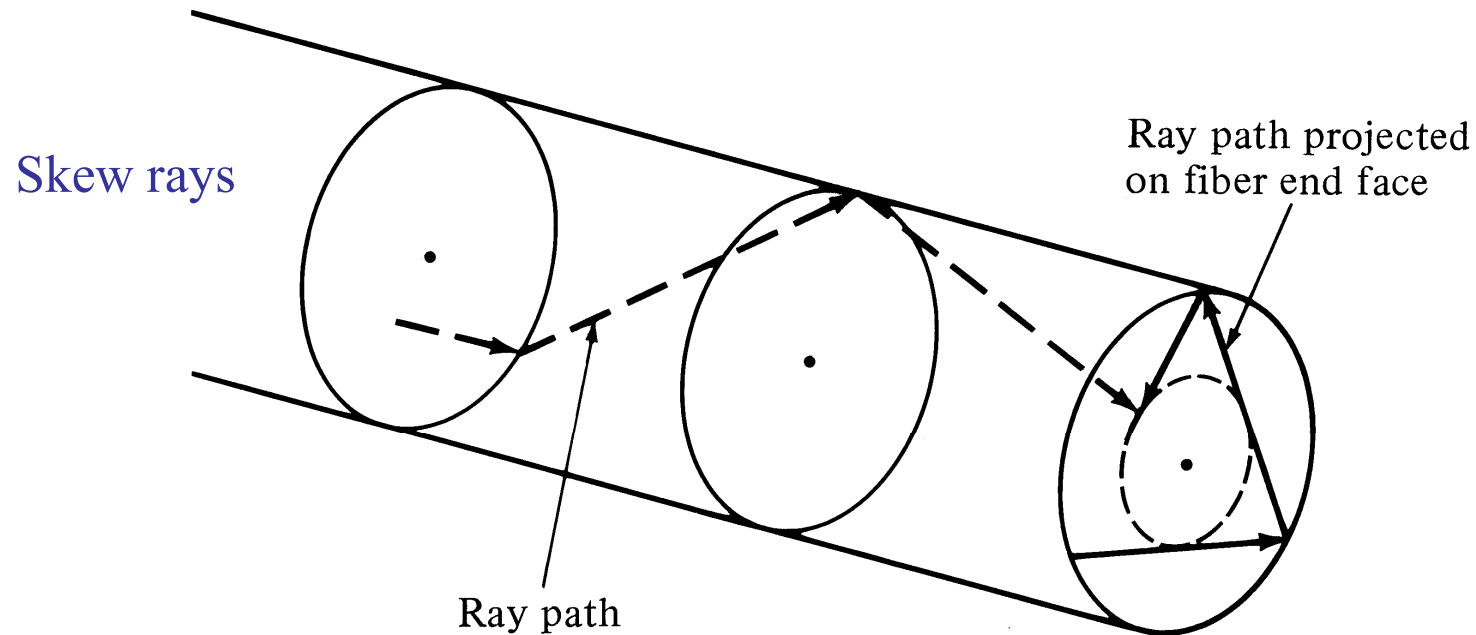
$r = 0$

r

Mode field diameter

E_0

$e^{-2}E_0$

Mode
field
diameter
$2W_0$

y

x

n_x

Horizontal mode

y

x

n_y
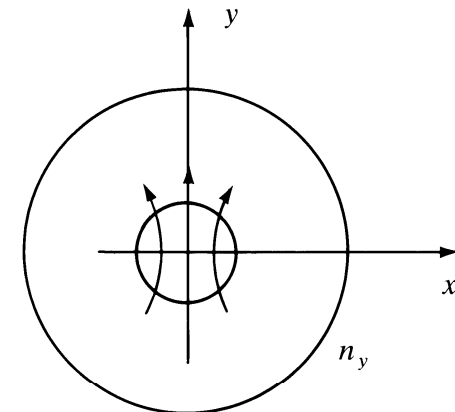
Vertical mode

Polarizations of fundamental mode

Ray Optics Theory (Step-Index Fiber)

Skew rays

Ray path projected
on fiber end face

Ray path

Each particular guided mode in a fiber can be represented by a group of rays which
Make the same angle with the axis of the fiber.

Different Structures of Optical Fiber

Index Profile

Fiber Cross Section and Ray Paths

Typical Dimensions

$n_2|$ $|n_1$

$2a$

125 μm
(cladding)

8–12 μm
(core)

Monomode step-index fiber

$n_2|$ $|n_1$

$2a$

125–400 μm
(cladding)

50–200 μm
(core)

Multimode step-index fiber

$n_2|$ $|n_1$

$r = a$

r $r = 0$

125–140 μm
(cladding)

50–100 μm
(core)

Multimode graded-index fiber

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000
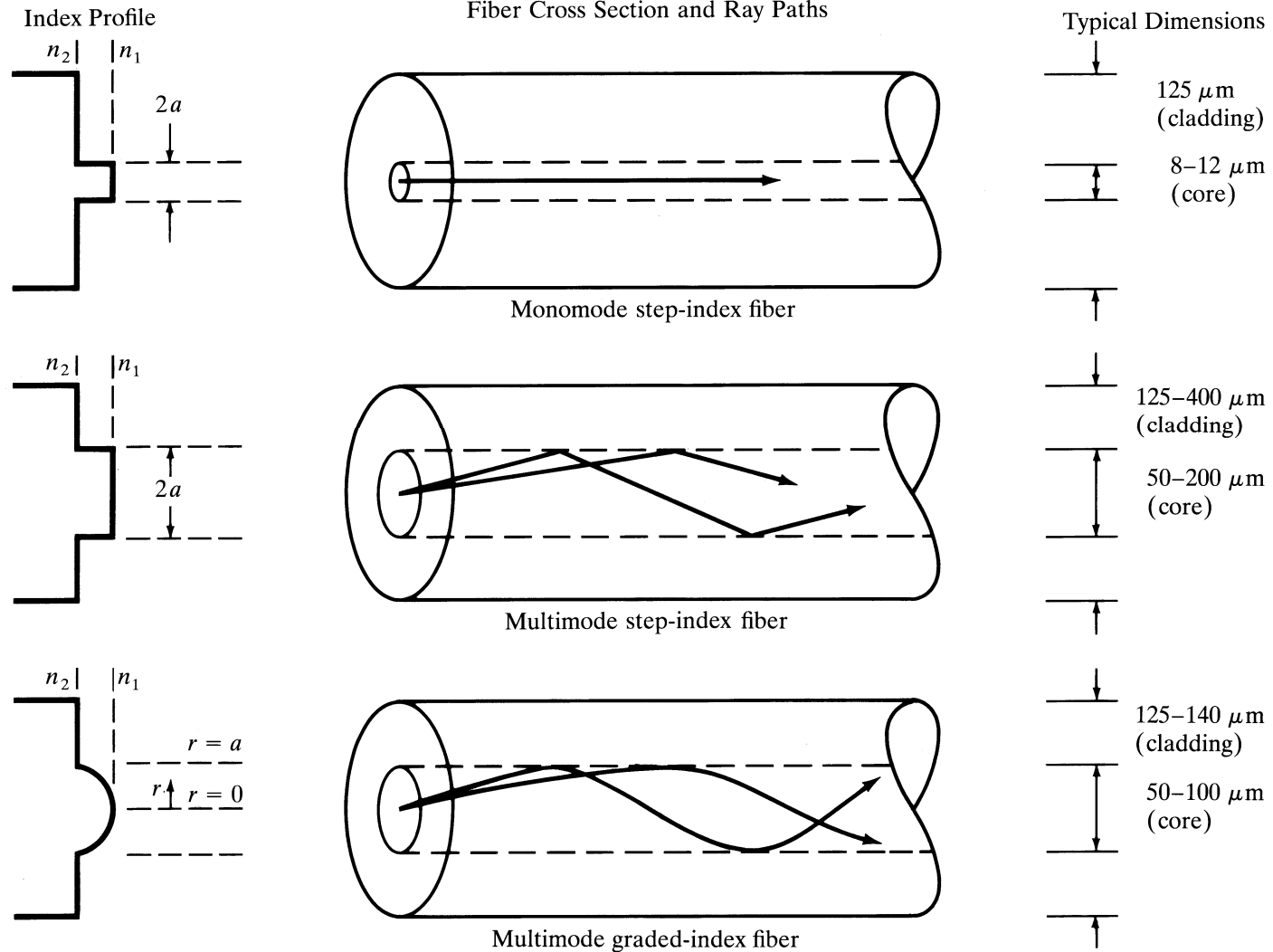
Mode designation in circular cylindrical waveguide (Optical Fiber)

TE_{lm} modes : The electric field vector lies in transverse plane.
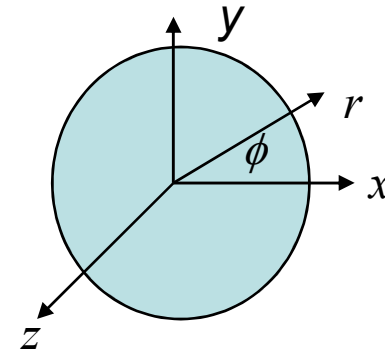
TM_{lm} modes : The magnetic field vector lies in transverse plane.

Hybrid HE_{lm} modes :TE component is larger than TM component.

Hybrid EH_{lm} modes : TM component is larger than TE component.

$l=$ # of variation cycles or zeros in ϕ direction.
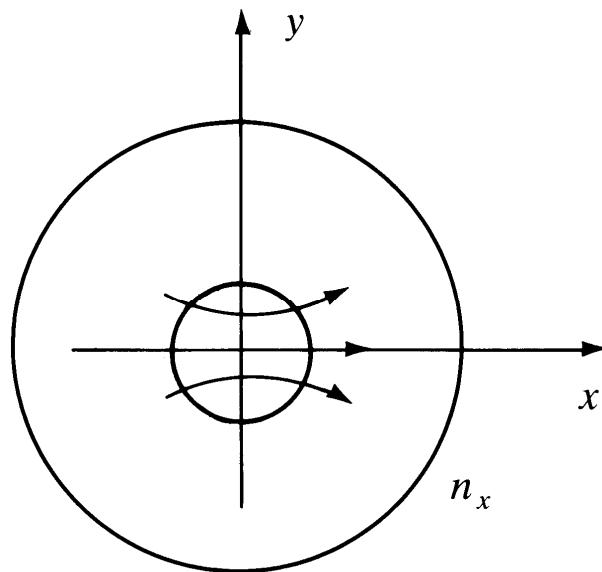$m=$ # of variation cycles or zeros in r direction.

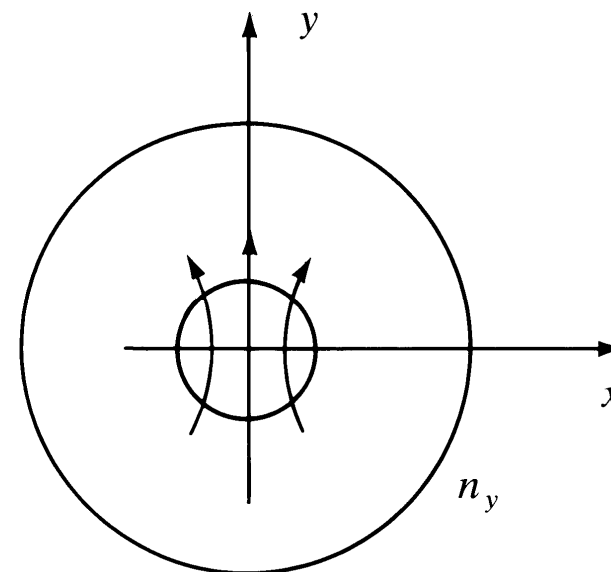**Linearly Polarized (LP) modes** in weakly-guided fibers ($n_1 - n_2 \ll 1$)

$\text{LP}_{0m} (\text{HE}_{1m}), \text{LP}_{1m} (\text{TE}_{0m} + \text{TM}_{0m} + \text{HE}_{0m})$

Fundamental Mode: $\text{LP}_{01} (\text{HE}_{11})$

Two degenerate fundamental modes in Fibers (Horizontal & Vertical HE_{11} Modes)


Horizontal mode

Vertical mode

Mode propagation constant as a function of frequency

- Mode propagation constant, $\beta_{lm}(\omega)$ is the most important transmission characteristic of an optical fiber, because the field distribution can be easily written in the form of eq. [2-27].

- In order to find a mode propagation constant and cut-off frequencies of various modes of the optical fiber, first we have to calculate the **normalized frequency**, V, defined by:

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda}\text{NA} \qquad \text{[2-30]}$$

a: radius of the core, λ is the optical free space wavelength, $n_1 \& n_2$ are the refractive indices of the core & cladding.

Plots of the propagation constant as a function of normalized frequency for a few of the lowest-order modes
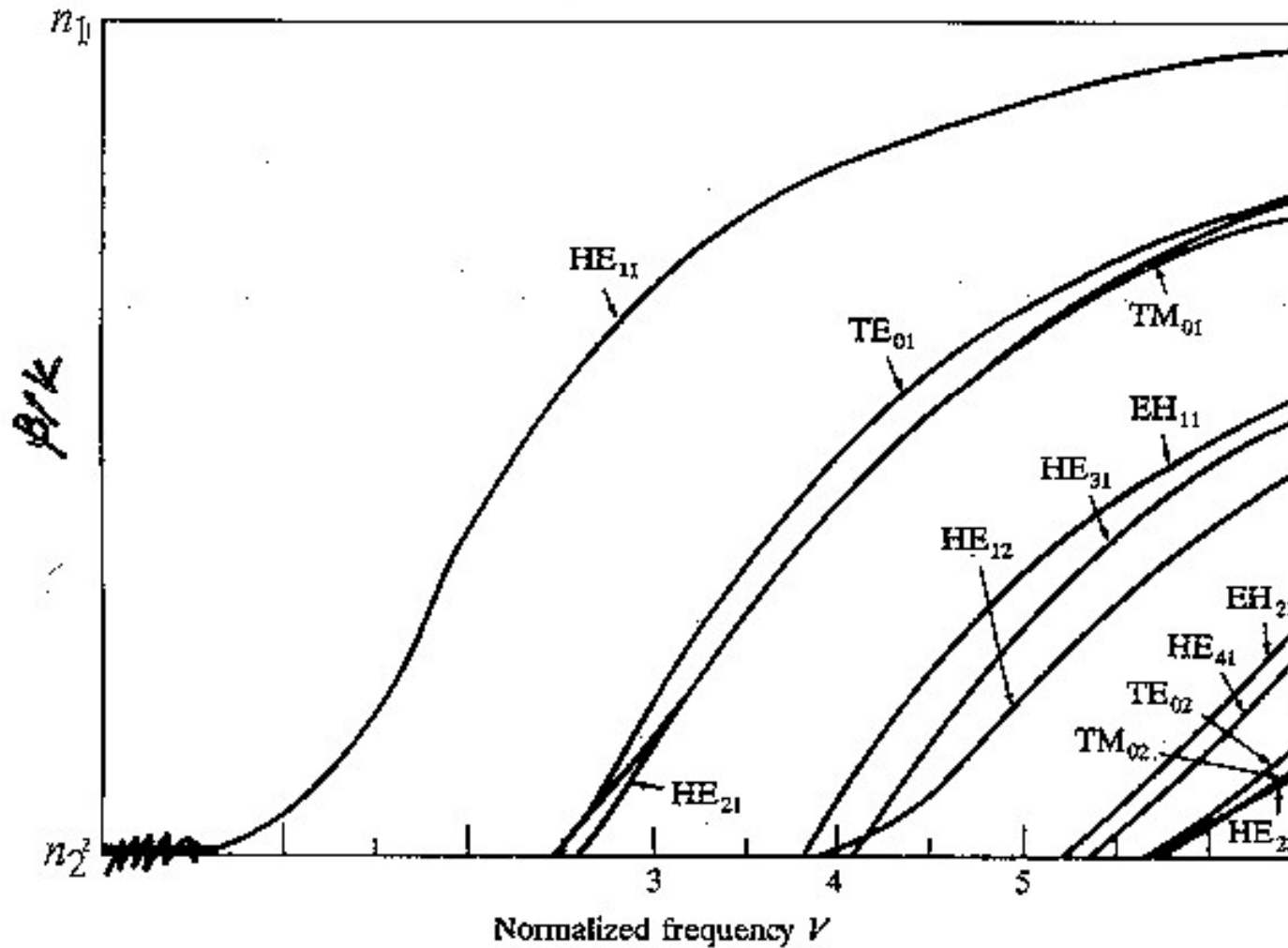
[Optical Fiber Communications, 3rd Edition, by Gerd Keiser, Mc Graw-Hill, 2000]

Single mode Operation

- The cut-off wavelength or frequency for each mode is obtained from:

$$\beta_{lm}(\omega_c) = n_2 k = \frac{2\pi n_2}{\lambda_c} = \frac{\omega_c n_2}{c} \qquad \text{[2-31]}$$

- **Single mode operation** is possible (Single mode fiber) when:

$$V \leq 2.405 \qquad \text{[2-32]}$$

Only HE_{11} can propagate faithfully along optical fiber

Single-Mode Fibers

$$\Delta = 0.1\% \text{ to } 1\% \quad ; \quad a = 6 \text{ to } 12 \ \mu\text{m} \ ;$$

$$V = 2.3 \text{ to } 2.4 \quad @ \text{ max frequency or min } \lambda$$

- **Example**: A fiber with a radius of 4 micrometer and $n_1 = 1.500 \ \& \ n_2 = 1.498$ has a normalized frequency of $V=2.38$ at a wavelength 1 micrometer. The fiber is single-mode for all wavelengths greater and equal to 1 micrometer.
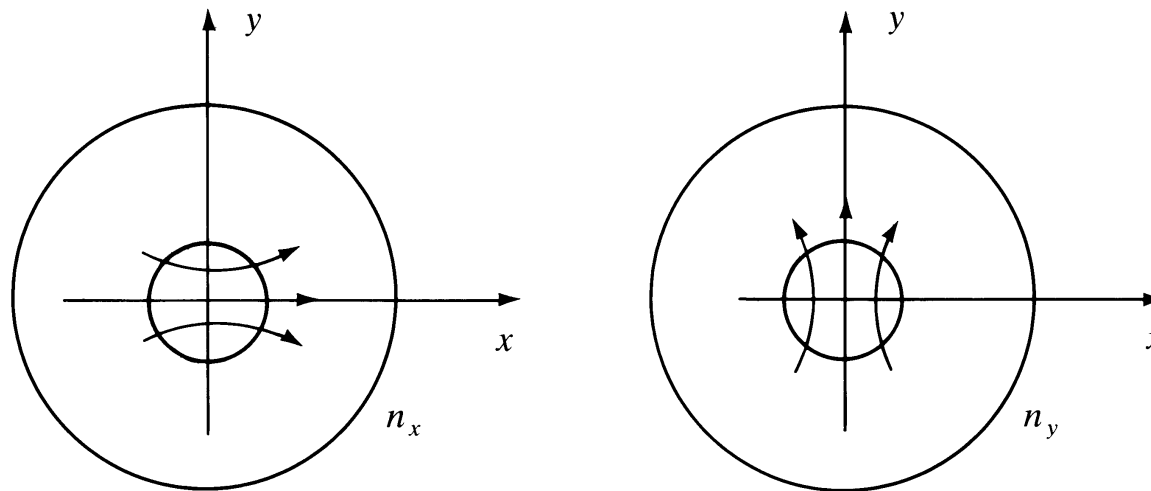
MFD (Mode Field Diameter): The electric field of the first fundamental mode can be written as:

$$E(r) = E_0 \exp(-\frac{r^2}{W_0^2}); \qquad \text{MFD} = 2W_0 \qquad\qquad \text{[2-33]}$$

Birefringence in single-mode fibers

- Because of asymmetries the refractive indices for the two degenerate modes (vertical & horizontal polarizations) are different. This difference is referred to as **birefringence**, B_f :

$$B_f = n_y - n_x \qquad \text{[2-34]}$$

Horizontal mode   Vertical mode

Fiber Beat Length

- In general, a linearly polarized mode is a combination of both of the degenerate modes. As the modal wave travels along the fiber, the difference in the refractive indices would change the phase difference between these two components & thereby the state of the polarization of the mode. However after certain length referred to as **fiber beat length,** the modal wave will produce its original state of polarization. This length is simply given by:

$$L_p = \frac{2\pi}{kB_f}$$

[2-35]

Multi-Mode Operation

- Total number of modes, M, supported by a multi-mode fiber is approximately (When V is large) given by:

$$M \approx \frac{V^2}{2} \qquad\qquad \text{[2-36]}$$

- **Power distribution in the core & the cladding:** Another quantity of interest is the ratio of the mode power in the cladding, P_{clad} to the total optical power in the fiber, P, which at the wavelengths (or frequencies) far from the cut-off is given by:

$$\frac{P_{clad}}{P} \approx \frac{4}{3\sqrt{M}} \qquad\qquad \text{[2-37]}$$